GRADUATE AERONAUTICAL LABORATORIES
CALIFORNIA INSTITUTE OF TECHNOLOGY

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of Conductive Heat Transfer from a Fine Wire

by

Lester Lees and Chung-Yen Liu

HYPERSONIC RESEARCH PROJECT
Memorandum No. 65
March 30, 1962
Army Ordnance Contract No. DA-04-495-ORD-3231

Firestone Flight Sciences Laboratory

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ABSTRACT

The Maxwell moment method utilizing the two-sided Maxwellian distribution function is applied to the problem of conductive heat transfer between two concentric cylinders at rest. Analytical solutions are obtained for small temperature differences between the cylinders. The predicted heat transfer agrees very well with experiments performed by Bomelburg, Schäfer-Rating and Eucken. Comparison with results given by Grad's thirteen moment equations, and with those given by Fourier's "law" plus the Maxwell-Smoluchowski temperature-jump boundary condition shows that the two-sided character in the distribution function is a crucial factor in problems involving surface curvature.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF SYMBOLS

a	thermal accommodation coefficient
A_2	= 1.3682, value of scattering integral
b	impact parameter
B	integration constant
$\overline{c}$	mean thermal velocity, $\sqrt{8kT/\pi m}$
f	velocity distribution function
f_o	local full-range Maxwellian
f_1, f_2	components of two-stream Maxwellian
F	interparticle force
G	function defined by the relation $\overline{n_1}\,\overline{T_1} + \overline{n_2}\,\overline{T_2}$
k	Boltzmann constant
k_c	"classical" thermal conductivity
K	quantity used in Welander's work (See Section I.), also, function defined by the relation $\overline{n_1}\,\overline{T_1} - \overline{n_2}\,\overline{T_2}$
$\tilde{K}$	constant in expression for inverse fifth-power force law, $F = (m_1\, m_2\, \tilde{K})/r^5$
ℓ	length of heated wire
m	mass of a particle
m_1 , m_2	mass of two interacting particles
n_1 , n_2	number density functions in two-stream Maxwellian
N_1 , N_2	perturbations of n_1 , n_2 over unity
p	nkT , hydrostatic pressure
P_{RR}, $P_{\theta\theta}$, P_{zz}	normal stresses in R, θ, Z directions
q_R	radial heat transfer rate
Q	arbitrary function of particle velocity, also total heat transfer from heated wire
ΔQ	change in Q produced by collisions
r	distance between two particles
R	radial distance
$\overline{R}_\ell$	non-dimensional radial distance of Knudsen layer from center of heated wire
R_1 , R_2	radii of inner and outer cylinders
ΔR_1, ΔR_2	free molecular heat transfer regions near R_1 and R_2
t_1 , t_2	perturbations of T_1 , T_2 over unity
T	absolute temperature
T_1 , T_2	temperature functions in two-stream Maxwellian
T_I , T_{II}	absolute temperatures of inner and outer cylinders

u_R	mean radial velocity
V	relative velocity between two interacting particles
z	axial distance
α	wedge angle, $\cos^{-1}(R_1/R)$
β	integration constant
δ	parameter defined by Eq. (17)
δ'	parameter defined by Eq. (23)
Δ	non-dimensional radial distance of Knudsen layer from surface of heated wire
ε	angle between plane of the orbit and plane containing the original relative velocity and the x-axis in a binary collision (See Reference 13.)
ϵ	$(T_I - T_{II})/T_I$
θ	circular angle in cylindrical coordinates
λ	Maxwell mean free path
Λ	correction factor used in Dickins' work (See Section I.)
μ_c	"classical" viscosity coefficient
$\vec{\xi}$	vector particle velocity
$d\vec{\xi}$	$d\xi_i\, d\xi_j\, d\xi_k$
$\vec{\xi}_P$	planar velocity vector, $\xi_p = \sqrt{\xi_R^2 + \xi_\theta^2}$
ξ_R, ξ_θ, ξ_z	velocity components of particle velocity in R, θ, Z directions
ρ	mean mass density
ϕ	angle between particle planar velocity $\vec{\xi}_p$ and radius vector $\vec{R}$

The subscripts "I" and "II" refer to quantities given at the inner and outer cylinders, respectively. The subscript "∞" denotes quantities evaluated at the continuum limit. The bar (—) superscript refers to non-dimensional quantities.

I. DESCRIPTION OF THE PROBLEM

The present problem deals with the conductive heat transfer from a metallic wire to a monatomic gas at rest. A fine wire is placed coaxially in a large cylindrical bell jar and is electrically heated. The wire temperature is known from its electrical resistance, while the heat input is found by measuring the current. At normal gas density, heat conduction from the wire is clearly independent of the gas pressure; while at very low gas density the heat loss is proportional to gas pressure. When the gas density is in the transition range, the relation between heat conduction and pressure is not as simple, but the two limiting regimes are joined smoothly (Figure 1). This simple device has long been used by many investigators[1, 2, 3, 4, 6] to determine the thermal conductivity of gases and to study the phenomena of temperature jump and energy accommodation at the wire surface. Some authors have also approached the problem analytically, but they are all forced to introduce certain ad hoc assumptions, which restrict their results to small values of the ratio of the mean free path to the wire radius.

It is easy to see that this type of instrument enjoys the privilege of simplicity. Up until very recently experiments with such a heated wire furnished one of the few sets of data for the full range of gas densities from the free molecular regime to the continuum regime. Moreover, the present problem is fundamentally important because it contains the effect of both convex and concave surfaces. The effects of curvature on heat transfer and temperature distribution in rarefied gases have never been thoroughly investigated. Along with plane Couette flow and shock wave structure, this problem has received a good deal of attention in rarefied gas dynamics. Weber[1] and Schäfer-Rating, and Eucken[2] sub-divide the annulus into three parts: two free molecular heat transfer regions near the solid surfaces $R_1 < R < R_1 + \Delta R_1$, $R_2 - \Delta R_2 < R < R_2$ (R_1, R_2 are the wire and bell jar radii respectively); and a region between $R_1 + \Delta R_1$ and $R_2 - \Delta R_2$ where continuum heat conduction is assumed. The arbitrary quantities ΔR_1 and ΔR_2 are functions of the mean free path λ, and in Weber's case they are taken simply equal to $(15/8)\lambda$. In Schäfer-Rating and Eucken's calculation $\Delta R_1/\lambda$ and $\Delta R_2/\lambda$ are functions of λ/R_1 and λ/R_2, respectively. In their studies, the implication that λ is small in comparison with R_1 and R_2 has been made.

The choice of ΔR_1 and ΔR_2 is clearly related to the temperature jump boundary condition proposed by Smoluchowski (See Section IV. 4.)[5]. He suggested that for a small degree of rarefaction, the difference between the gas temperature and wall temperature at the solid surface is equal to $-(15/8)\ \lambda\ (dT/dn)_{wall}$, where $(dT/dn)_{wall}$ is the gas temperature gradient normal to the wall. Application of Smoluchowski's relation to the present problem is discussed in Section IV. 4.

Gregory[3] and his followers have investigated this "hot-wire" method over the period of a decade. Their primary goal is accurate determination of the gaseous thermal conductivity as a function of temperature. Early developments were more along technical lines than analytical, like keeping the wire temperature constant under different conditions, elimination of convective losses, etc. In computation, they merely used the usual Fourier result that the total heat transfer Q is $2\pi\ k_c\ \ell\ (T_I - T_{II})\ /\ \ln\ (R_2/R_1)$, in which k_c is the "classical" thermal conductivity of the gas*, ℓ the length of the wire, and T_I and T_{II} are the temperatures of the wire and the bell jar, respectively. They allow k_c to decrease if the pressure decreases appreciably below atmospheric[3]; however, their original focal point is the temperature dependence but not the pressure effect.

Later, Dickins[4] adopted Gregory's apparatus to determine accommodation coefficients.** He corrected the heat transfer Q at low pressures by an amount Λ/R_1, so that

$$Q = \frac{2\pi\ k_c\ \ell\ (T_I - T_{II})}{\ln\ (R_2/R_1) + (\Lambda/R_1)} \quad ,$$

---

* It should be noted that the Fourier relation $\vec{q} = -\ k_c\ \vec{\nabla}\ T$ holds only at normal densities; thus the "classical" thermal conductivity k_c introduced here is merely for convenience. (See Section IV. 4.)

** The thermal accommodation coefficient "a" advanced by Knudsen[26] is defined as

$$a = \frac{E_g - E_r}{E_g - E_w}$$

E_g = energy transported to surface by incident molecules in equilibrium at the gas temperature
E_r = actual energy carried away by molecules leaving the surface
E_w = energy of re-emitted molecules in equilibrium at the wall temperature

in which $\Lambda = (15/8)\,\lambda\,(2-a)/a$, and a is Knudsen's accommodation coefficient. As determined by Dickins' experiment "a" is about 0.9 for most gases except helium and hydrogen. The correction Λ is easily seen to be based on Smoluchowski's relation[5].

Two years later, Gregory[6] generalized the same relation for polyatomic gases, but Λ then included a numerical factor which accounts for intermolecular forces and has to be determined by experimental data on viscosity and specific heat. At the same time, microscopic studies have also been made by Zener[7] and Devonshire[8] on the general aspect of solid-gas interchange of energy. They require experimental determination of certain constants related to intermolecular forces.

Welander in 1954[9] worked the problem anew but used a different constant for Λ, in which the factor $\frac{2-a}{a}$ is replaced by $\frac{2-Ka}{a}$.* The quantity K is found to be 0.827 by solving the "Krooked" Boltzmann equation[10]**, in which the collision integral is taken to be $(8/15)(\bar{c}/\lambda)(f - f_o)$, where f is the unknown velocity distribution function, f_o is the local Maxwellian, and $\bar{c}$ the mean thermal velocity $\bar{c} = \sqrt{(8kT/m\pi)}$. Welander attempted to extend the validity of the $\frac{2-Ka}{a}$ expression to the free molecular regime by allowing K to be a function of gas density. Under the assumption that $\left|(dT/dR)_{wall}\,(\lambda/T)\right|$ is small in comparison with unity and that the distribution function differs slightly from the local Maxwellian, he obtained an integral equation governing the K- function, but he did not solve that equation. Instead he estimated K from experimental data given by Schäfer-Rating and Eucken[2], and found that K varied between 0.1 and 0.6. The fact that K depends only on pressure is rather obvious; yet Welander's result demonstrates very little beyond this point.

Though it might be difficult to record all the investigations of this "simple" problem since the first use of the apparatus by Schleiermacher[11] for determination of gaseous conductivity in 1888, yet it is clear that a thorough theoretical investigation of the problem is long overdue.

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* Welander also used a different numerical factor $75\pi/128$ instead of 15/8; however, the quantative difference is negligible.

** Welander's paper is published at the same time as Krook's work[27].

II. FORMULATION OF THE PROBLEM ACCORDING TO THE MAXWELL MOMENT METHOD

II. 1. Distribution Function and Mean Quantities

We consider a wire of radius R_1 placed at the center of a concentric cylinder of radius R_2, with $R_2 > R_1$ (Figure 2). The wire is heated to a temperature T_I, while the outer cylinder is kept at temperature T_{II}. The annular region ($R_1 < R < R_2$) is filled with monatomic gas at an arbitrary density level, which is characterized by the mean free path λ evaluated at a convenient reference point (say $R = R_1$). If the wire is sufficiently long, end effects are negligible; thus the problem is axially symmetric and two-dimensional.

In compliance with the requirements given by Lees[13] (see also introduction, GALCIT Hypersonic Research Project, Memorandum No. 58), the simplest distribution function having a "two-sided" character and capable of giving a smooth transition between the highly rarefied gas regime and the continuum limit consists of two Maxwellians, each containing several parametric functions. All outwardly directed molecules with planar velocity vector $\vec{\xi}_p$ ($\xi_p = \sqrt{\xi_R^2 + \xi_\theta^2}$, $\phi = \tan^{-1}(\xi_R/\xi_\theta)$) lying inside the wedge of influence (region I in Figure 2) are characterized by one Maxwellian f_1, where

$$f = f_1 \quad \text{for} \quad \alpha < \phi < \pi - \alpha$$

in which

$$\alpha = \cos^{-1}(R_1/R) \quad .$$

Then, all molecules with planar velocity $\vec{\xi}_p$ lying outside of region I are characterized by f_2, i.e.,

$$f = f_2 \quad \text{for} \quad \pi - \alpha < \phi < 2\pi + \alpha \quad .$$

The requirement that f should be discontinuous on the sides of the "wedge of influence" is the most basic feature in the present scheme; its importance will be seen shortly.

In order to satisfy at least the three conservation equations and the heat flux equation, one finds that four parametric functions specifying f_1 and f_2 are the absolute minimum. Thus we prescribe that

$$f_1 = n_1 \left(\frac{m}{2\pi k T_1} \right)^{3/2} \exp. \left[- \frac{m}{2kT_1} \left(\xi_p^2 + \xi_z^2 \right) \right] \quad ;$$

likewise;

$$f_2 = n_2 \left(\frac{m}{2\pi k T_2} \right)^{3/2} \exp. \left[- \frac{m}{2kT_2} \left(\xi_p^2 + \xi_z^2 \right) \right]$$

where $n_1(R)$, $T_1(R)$, $n_2(R)$, $T_2(R)$ are the four unknown functions of radial distance. Here the n's have the dimension of a number density, while the T's have the dimension of a temperature, and it must be stressed that each individual function has no explicit physical significance in general.

Knowing the distribution function f, one can evaluate all mean quantities $\tilde{\phi}$ by averaging over all velocity space,

$$\langle \tilde{\phi} \rangle = \int \tilde{\phi} f d\vec{\xi} = \int_{\alpha}^{\pi-\alpha} \int_{0}^{\infty} \int_{-\infty}^{+\infty} \tilde{\phi} f_1 \, d\xi_z \, d\xi_p \, d\phi + \int_{\pi-\alpha}^{2\pi+\alpha} \int_{0}^{\infty} \int_{-\infty}^{+\infty} \tilde{\phi} f_2 \, d\xi_z \, d\xi_p \, d\phi \quad .$$

For example, the mean density is

$$\rho = \rho(R) = \int m f \, d\vec{\xi} = (m/2\pi) \left[n_1(\pi - 2\alpha) + n_2(\pi + 2\alpha) \right] \tag{1}$$

and the mean temperature is

$$T = T(R) = \frac{n_1 T_1 (\pi - 2\alpha) + n_2 T_2 (\pi + 2\alpha)}{n_1(\pi - 2\alpha) + n_2 (\pi + 2\alpha)} \tag{2}$$

Notice that the angular dependence appears directly, while T_1 , T_2 , n_1 , n_2 will bring in a purely radial dependence. Expressions for radial velocity u_R , hydrostatic pressure p, and radial heat transfer q_R are listed below for later usage:

$$u_R = \sqrt{(2\pi k/m)} \cdot (\cos . \alpha) . \frac{n_1 \sqrt{T_1} - n_2 \sqrt{T_2}}{n_1 (\pi - 2\alpha) + n_2 (\pi + 2\alpha)} \tag{3}$$

$$p = (k/2\pi) \left[n_1 T_1 (\pi - 2\alpha) + n_2 T_2 (\pi + 2\alpha) \right] = - \frac{P_{RR} + P_{\theta\theta} + P_{zz}}{3} \tag{4a}$$

$$\left.
\begin{aligned}
P_{RR} &= - \langle \rho \xi_R^2 \rangle = - \frac{k}{2\pi} \left[n_1 T_1 (\pi - 2\alpha + \sin 2\alpha) + n_2 T_2 (\pi + 2\alpha - \sin 2\alpha) \right] \\
P_{\theta\theta} &= - \langle \rho \xi_\theta^2 \rangle = - \frac{k}{2\pi} \left[n_1 T_1 (\pi - 2\alpha - \sin 2\alpha) + n_2 T_2 (\pi + 2\alpha + \sin 2\alpha) \right] \\
P_{zz} &= - \langle \rho \xi_z^2 \rangle = - \frac{k}{2\pi} \left[n_1 T_1 (\pi - 2\alpha) + n_2 T_2 (\pi + 2\alpha) \right]
\end{aligned}
\right\} \tag{4b}$$

$$q_R = \sqrt{(2/\pi m)} \cdot (\cos \cdot \alpha)\left[n_1 (kT_1)^{3/2} - n_2 (kT_2)^{3/2}\right] \qquad (5)$$

It should be pointed out here that the normal stresses in different directions are generally not the same; namely, $P_{RR} \neq P_{\theta\theta} \neq -p$.

II. 2. Differential Equations

In cylindrical coordinates the Maxwell integral equation of transfer is as follows[13]:

$$\frac{1}{R}\frac{\partial}{\partial R}\left[R\int f \xi_R Q \, d\vec{\xi}\right] + \frac{1}{R}\frac{\partial}{\partial \theta}\int f \xi_\theta Q \, d\vec{\xi} + \frac{\partial}{\partial z}\int f \xi_z Q \, d\vec{\xi} + - \int \frac{f}{R}\left(\xi_\theta^2 \frac{\partial Q}{\partial \xi_R} - \xi_\theta \xi_R \frac{\partial Q}{\partial \xi_\theta}\right) d\vec{\xi} = \Delta Q \qquad (6)$$

where

$$Q = Q(\xi_R, \xi_\theta, \xi_z) = Q(\xi_p, \phi, \xi_z)$$

$$\Delta Q = \iiint (Q' - Q)\, f f_1 \, V d\vec{\xi}\, d\vec{\xi}_1 \, b \, d \, b \, d\varepsilon \quad *$$

Because of two-dimensionality and axial-symmetry, Eq. (6) further reduces to

$$\frac{1}{R}\frac{d}{dR}\left[R\int f \xi_R Q \, d\vec{\xi}\right] - \int \frac{f}{R}\left(\xi_\theta^2 \frac{\partial Q}{\partial \xi_R} - \xi_\theta \xi_R \frac{\partial Q}{\partial \xi_\theta}\right) d\vec{\xi} = \Delta Q, \qquad (7)$$

Setting $Q = m$, $m\xi_R$, $m\xi^2/2$, respectively, we find $\Delta Q = 0$ because the mass, momentum, and energy are invariant during collisions, and we obtain the ordinary continuity, radial momentum, and energy equations. Since we are primarily interested in radial heat transfer, we take $Q_4 = m\xi_R \xi^2/2$, which yields the heat flux equation in which the collision integral ΔQ , for simplicity, is evaluated with Maxwell's inverse fifth power force law $F = \dfrac{m_1 m_2 \tilde{K}}{r^5}$, and is found to be proportional to the heat flux q_R[13] . The four differential equations governing the four unknown functions are as follows:

Continuity

$$\left[Q = m\right], \qquad n_1 \sqrt{T_1} = n_2 \sqrt{T_2} \qquad (8a)$$

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* See Introduction of Reference 15; also see Reference 13.

R-Momentum

$$\left[Q = m\xi_R\right] \quad , \tag{8b}$$

$$(\sin 2\alpha - 2\alpha)\,(d/dR)(n_1T_1 - n_2T_2) + \pi\,(d/dR)\,(n_1T_1 + n_2T_2) = 0$$

Energy

$$\left[Q = (m/2)\xi^2\right] \quad ,$$

$$(\cos\,.\,\alpha)(n_1T_1^{3/2} - n_2T_2^{3/2}) = B/R \tag{8c}$$

Heat Flux

$$\left[Q = (m/2)\,\xi_R\xi^2\right] \quad ,$$

$$(\sin 2\alpha - 2\alpha)\,(d/dR)(n_1T_1^2 - n_2T_2^2) + \pi\,(d/dR)\,(n_1T_1^2 + n_2T_2^2)$$

$$= -\,(4/5)\,m\,A_2\,\sqrt{\widetilde{K}/k\pi}\;\;.\;(B/R)\left[n_1(\pi-2\alpha)+n_2(\pi+2\alpha)\right] \tag{8d}$$

In Eq. (8c), B is an undetermined integration constant; in Eq. (8d), $A_2 \equiv 1.3682$ is the value of the scattering integral for Maxwell molecules[14], and k is the Boltzmann constant. A_2 and $\widetilde{K}$ are related to the "classical" coefficient of viscosity by the expression

$$\mu_c = \frac{k\,T}{(3/2)\,A_2\sqrt{2\,m\,\widetilde{K}}} \quad . \quad *$$

Since $P_{RR} \neq P_{\theta\theta} \neq -p$ the momentum equation, Eq. (8b) does not imply $(dp/dR) = 0$. This observation is important, because a pressure gradient exists owing to heat conduction but not because of fluid flow. Also, the heat flux equation, Eq. (8d), relating q_R to two higher moments:

$$\int m f\;\xi_p^2 \sin^2\phi\,(\xi_p^2 + \xi_z^2)\,d\vec{\xi} \quad \text{and} \quad \int m f \xi_p^2 \cos^2\phi\,(\xi_p^2 + \xi_z^2)d\vec{\xi}$$

bears no resemblence to Fourier's "law" in general; in fact Eq. (8d) reduces to $q_R = -\,k_c\,(dT/dR)$ only if the local full-range Maxwellian is introduced into the left-hand side. In other words, Eq. (8d) would give Fourier's "law" to the first order if the Chapman-Enskog expansion procedure is employed[13].

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* As it has been mentioned in Reference 15,

$\mu_c \neq p_{jk} \;/\; \left(\frac{\partial u_j}{\partial x_k} + \frac{\partial u_k}{\partial x_j}\right)$ except in the Navier-Stokes regime.

Also it should be pointed out that the Prandtl number for Maxwell molecules is equal to 2/3.

II. 3. Boundary Conditions

For completely diffusive reemission, the boundary conditions are very simple[13], namely

$$T_1 = T_I \quad \text{at} \quad R = R_1 \text{ (Figure 2)} \tag{9a}$$

$$T_2 = T_{II} \quad \text{at} \quad R = R_2 \tag{9b}$$

One additional condition is to specify the density level at a convenient point. We may set

$$n_1 = n_I \quad \text{at} \quad R = R_1 \quad . \tag{9c}$$

The fourth condition is the vanishing of mean radial velocity $u_R = 0$ at $R = R_1$ and $R = R_2$. From the expression for u_R [Eq. (3)] and the continuity equation [Eq. (8a)], we conclude immediately that $u_R \equiv 0$ everywhere in the annulus, or

$$n_1 \sqrt{T_1} = n_2 \sqrt{T_2} \tag{9d}$$

These four boundary conditions are sufficient for the four equations [Eqs. (8)].

II. 4. Differential Equations and Boundary Conditions in Non-Dimensional Form

In order to bring out all pertinent parameters governing the problem, we normalize Eqs. (8) and (9) by choosing n_I, T_I, R_1 as the characteristic number density, temperature, and length, respectively. We also utilize the fact that the Maxwell mean free path evaluated at condition I is

$$\lambda_I = \frac{1}{3A_2 \rho_I} \sqrt{\frac{\pi k T_I}{\tilde{K}}} \quad .$$

Denoting all normalized quantities by a bar superscript, like

$$\bar{n}_1 = (n_1/n_I) \quad , \quad \bar{T} = (T/T_I) \quad , \quad \ldots\ldots$$

and so on, Eqs. (8) in non-dimensional form are as follows:

Continuity

$$\bar{n}_1 \sqrt{\bar{T}_1} = \bar{n}_2 \sqrt{\bar{T}_2} \tag{10a}$$

R-Momentum

$$(\sin 2\alpha - 2\alpha)\frac{d}{d\bar{R}}(\overline{n_1}\,\overline{T_1} - \overline{n_2}\,\overline{T_2}) + \pi\frac{d}{d\bar{R}}(\overline{n_1}\,\overline{T_1} + \overline{n_2}\,\overline{T_2}) = 0 \qquad (10b)$$

Energy

$$(\overline{n_1}\,\overline{T_1}^{3/2} - \overline{n_2}\,\overline{T_2}^{3/2}) = \beta \qquad (10c)$$

Heat Flux

$$(\sin 2\alpha - 2\alpha)\frac{d}{d\bar{R}}(\overline{n_1}\,\overline{T_1}^{2} - \overline{n_2}\,\overline{T_2}^{2}) + \pi\frac{d}{d\bar{R}}(\overline{n_1}\,\overline{T_1}^{2} + \overline{n_2}\,\overline{T_2}^{2}) + (4/15)\cdot(R_1/\lambda_I)\cdot(\beta/\bar{R})\left[\overline{n}_1(\pi-2\alpha) + \overline{n}_2(\pi+2\alpha)\right] = 0 \qquad (10d)$$

in which β is the integration constant.

The normalized boundary conditions corresponding to Eqs. (9) are at $\bar{R} = 1$,

$$\overline{T_1} = 1 \qquad (11a)$$

$$\overline{n_1} = 1 \qquad (11c)$$

At $\bar{R} = (R_2/R_1)$,

$$\overline{T_2} = (T_{II}/T_I) \qquad (11b)$$

There are three parameters governing this problem: the rarefaction parameter λ_I/R_1 of Eq. (10d); the temperature ratio T_{II}/T_I appearing in the boundary condition; and the radius ratio R_2/R_1 describing the geometrical configuration. One can readily see that Eqs. (10) would all become algebraic at the free molecular limit, namely, $(\lambda_I/R_1) \longrightarrow \infty$; thus, $\overline{n_1}$, ... , $\overline{T_2}$ would all have the constant values prescribed by the boundary conditions. Then the distribution function f would not only be discontinuous in velocity space, but also independent of space coordinates. Nevertheless, mean quantities [See Eqs. (1-5).] would still depend on $\bar{R}$ even in free molecular flow. It has been mentioned previously that the set of equations, Eqs. (10), reduce to the usual Fourier formulation if an expansion in λ_I/R_1 is employed. Of course the complete solutions to these equations will demonstrate these limiting characteristics.

III. SOLUTIONS FOR SMALL TEMPERATURE DIFFERENCES BETWEEN CYLINDERS

In general, one can utilize Eqs. (10a), (10c) to express $\overline{n_1}$, $\overline{n_2}$, $\overline{T_1}$, $\overline{T_2}$ in favor of two unknown functions; as in Reference 15; then one has to integrate Eqs. (10b) and (10d) numerically for these two functions. For example, if we designate

$$\overline{n_1}\,\overline{T_1} + \overline{n_2}\,\overline{T_2} = G(\overline{R})$$

$$\overline{n_1}\,\overline{T_1} - \overline{n_2}\,\overline{T_2} = K(\overline{R})$$

then by using Eqs. (10a) and (10c), we can express all four unknown functions by these two new functions G and K as

$$\overline{n_1} = \frac{2}{\beta^2}\cdot\frac{KG}{\left(\frac{1}{K}+\frac{1}{G}\right)} \qquad \overline{n_2} = \frac{2}{\beta^2}\cdot\frac{KG}{\left(\frac{1}{K}-\frac{1}{G}\right)}$$

$$\overline{T_1} = \frac{\beta^2}{4}\cdot\left(\frac{1}{K}+\frac{1}{G}\right)^2 \qquad \overline{T_2} = \frac{\beta^2}{4}\cdot\left(\frac{1}{K}-\frac{1}{G}\right)^2 .$$

Substituting into Eqs. (10b) and (10d), we obtain two governing equations

$$\frac{dG}{d\overline{R}} = \frac{2\alpha - \sin 2\alpha}{\pi}\cdot\frac{dK}{d\overline{R}} = f_n(K, \overline{R})$$

$$\frac{dK}{d\overline{R}} = \frac{\left(\frac{32}{15}\cdot\frac{1}{\beta^3}\cdot\frac{R_1}{\lambda_I}\right)\cdot\left(\frac{\pi G - 2\alpha G}{1/K^2 - 1/G^2}\right)\cdot\frac{1}{\overline{R}}}{2(2\alpha - \sin 2\alpha)\left(\frac{1}{G^2}-\frac{1}{K^2}\right) + \pi\frac{G}{K^2} - \frac{K}{G^3}\cdot\frac{(\sin 2\alpha - 2\alpha)^2}{\pi}} = f_n(K, G, \overline{R})$$

Boundary conditions can be converted easily into conditions for G and K. As in all two-point boundary value problems, to start integration at one point one must make a guess on some undetermined constants, then adjust the guessed values until the boundary conditions at the other point are satisfied. For the present case, once the value of β is assumed, G and K at $\overline{R} = 1$ are known, integration can then be started from $\overline{R} = 1$ towards $\overline{R} = R_2/R_1$. The correct value of β would be the one that leads to the correct $\overline{T}_2$ value (i.e., $\overline{T}_2 = T_{II}/T_I$) at $\overline{R} = R_2/R_1$. In actual computation, interpolation would be more practical than the iteration scheme; namely, for a given R_1/λ_I, one may work with a spectrum of β's which leads to a spectrum of corresponding $\overline{T}_{II}$'s. Then for a prescribed $\overline{T}_{II}$, the corresponding β value at that particular density level can be found by interpolation.

By examining the situation more closely, one finds that the linear problem is in fact the most important. In all experiments previously performed, the wire is only slightly heated and its temperature never exceeds the temperature of the bell jar by more than 15 per cent. With large temperature difference pure conduction would be quite difficult to achieve. Thus, Eqs. (10) are linearized in order to acquire analytical solutions, to compare with experiments, and to study particularly the effect of surface curvature.

When the temperature ratio T_{II}/T_I departs little from unity, the four functions $\overline{n_1}$, $\overline{n_2}$, $\overline{T_1}$, $\overline{T_2}$ also depart from unity by an amount small compared with one. Symbolically, if

$$T_{II}/T_I = 1 - \epsilon \qquad \text{where} \quad \epsilon << 1 ,$$

then

$$\begin{aligned} \overline{n_1} &= 1 + N_1 \quad , & \overline{n_2} &= 1 + N_2 \\ \overline{T_1} &= 1 + t_1 \quad , & \overline{T_2} &= 1 + t_2 \end{aligned} \tag{12}$$

in which N_1 , N_2 , t_1 , $t_2 << 1$.

Such a limiting process implies that each of the distribution functions f_1 and f_2 are slightly perturbed over a constant Maxwellian, but the distribution function is still discontinuous on the surface of the wedge of influence (Figure 2). This procedure is intrinsically different from perturbation over a full-range local Maxwellian which usually is space dependent; the latter procedure follows practically the same line as the Chapman-Enskog scheme. Thus it should not be surprising to learn that the results so obtained would be useful only when the gas is slightly rarefied. In other words, the present linearization implies no restriction on the value of the rarefaction parameter. The scheme does imply that N_1 , N_2 , t_1 , t_2 are of the same order of magnitude.

Introducing Eqs. (12) in Eqs. (10), one readily finds the set of governing equations for quantities N_1 , N_2 , t_1 , t_2 .

$$N_1 + \tfrac{1}{2} t_1 = N_2 + \tfrac{1}{2} t_2 \tag{13a}$$

$$(\sin 2\alpha - 2\alpha) \frac{d}{d\overline{R}} (N_1 + t_1 - N_2 - t_2) + \pi \frac{d}{d\overline{R}} (N_1 + t_1 + N_2 + t_2) = 0 \tag{13b}$$

$$(N_1 - N_2) + (3/2)(t_1 - t_2) = \beta \quad . \tag{13c}$$

By using Eqs. (13a) and (13c), one obtains

$$t_1 - t_2 = \beta \qquad (13c')$$

and

$$N_2 - N_1 = \beta/2 \quad . \qquad (13a')$$

Eq. (13b) then yields

$$N_1 + t_1 + N_2 + t_2 = \text{constant}$$

or

$$N_1 + t_1 = \text{constant} \quad . \qquad (13b')$$

The heat flux equation Eq. (10d) becomes

$$(\sin 2\alpha - 2\alpha)\frac{d}{d\bar{R}}(N_1+2t_1-N_2-2t_2) + \pi\frac{d}{d\bar{R}}(N_1+2t_1+N_2+2t_2)+\frac{4}{15}\frac{R_1}{\lambda_I}\frac{\beta}{\bar{R}}(2\pi) = 0 \qquad (13d)$$

or [Eqs. (13a'b'c')]

$$\frac{d}{d\bar{R}}(t_1 + t_2) + (8/15)(R_1/\lambda_I)(\beta/\bar{R}) = 0$$

Integrating and applying the boundary conditions

$$\left.\begin{aligned} t_1 &= 0 \\ N_1 &= 0 \end{aligned}\right\} \quad \text{at } \bar{R} = 1 \qquad (14a), (14b)$$

$$t_2 = -\epsilon \quad \text{at } \bar{R} = (R_2/R_1) = \bar{R}_2 \quad . \qquad (14c)$$

One obtains the following solutions

$$\left.\begin{aligned} N_1 &= (4/15)\cdot(R_1/\lambda_I)\,\beta\,\ln\bar{R} \\ N_2 &= \beta\left(\tfrac{1}{2} + \frac{4}{15}\cdot\frac{R_1}{\lambda_I}\cdot\ln\bar{R}\right) \\ t_1 &= -(4/15)\cdot(R_1/\lambda_I)\,\beta\,\ln\bar{R} \\ t_2 &= -\beta\left[1 + (4/15)\cdot(R_1/\lambda_I)\ln\bar{R}\right] \end{aligned}\right\} \qquad (15)$$

with

$$\beta = \frac{\epsilon}{1 + (4/15)(R_1/\lambda_I)\ln(R_2/R_1)}$$

Using subscript ∞ to denote quantities evaluated at the continuum limit, we then find the heat transfer ratio as

$$\frac{Q}{Q_\infty} = \frac{q_R}{q_{R\infty}} = \frac{\beta}{\beta_\infty} = \frac{1}{1 + \dfrac{1}{\frac{4}{15}\frac{R_1}{\lambda_I}\ln\frac{R_2}{R_1}}} \qquad (16)$$

Inserting the results of N_1, N_2, t_1, t_2 into Eq. (2), we obtain the temperature distribution

$$\bar{T} = 1 - \epsilon \frac{\pi \left(1 + \frac{8}{15}\frac{R_1}{\lambda_I} \ln \frac{R}{R_1}\right) + 2\cos^{-1}\left(\frac{R_1}{R}\right)}{\left(1 + \frac{4}{15}\frac{R_1}{\lambda_I} \ln \frac{R_2}{R_1}\right)} + O(\epsilon^2)$$

or

$$\frac{1 - \bar{T}}{1 - \bar{T}_{II}} = \delta \left[\frac{1}{2} + \frac{1}{\pi} \cos^{-1}\left(\frac{R_1}{R}\right)\right] + (1 - \delta) \frac{\ln\left(\frac{R}{R_1}\right)}{\ln\left(\frac{R_2}{R_1}\right)} \tag{17}$$

with

$$\delta = \left[1 + (4/15)\,(R_1/\lambda_I)\,\ln\,(R_2/R_1)\right]^{-1} .$$

Other mean quantities can be immediately written down from Eqs. (1, 3, 4, 5, and 15) in a similar fashion. Results will be discussed in Section IV.

IV. DISCUSSION

IV. 1. Heat Transfer and Comparison with Experiment

Knudsen's formula for heat loss from a surface of temperature T_I to a stream of incident molecules of temperature T_{II} at low pressure[21] has been generally accepted as a good one:

$$q_R \Big|_{\text{at surface}} = \sqrt{\frac{2k}{\pi m}} \cdot \frac{pa}{\sqrt{T_I}} \cdot (T_I - T_{II}) \tag{18}$$

where a is Knudsen's thermal accommodation coefficient, which in the present study has been taken to be unity. Studies on accommodation coefficients, though quantitatively inconclusive, leave no doubt about the validity of Eq. (18). Dickins[4] observed the linear dependency of thermal conductivity on pressure below 5 cm Hg. Mann[22] also confirmed that "a" is independent of pressure within 2 ~ 3 per cent accuracy up to 330 microns for an instrument with $R_2/R_1 = 1250$. Of course direct measurements on heat loss support this fact quantitatively (Section I).

Owing to its "two-sided" character the present formulation naturally brings out Eq. (18) as a limit for $R_I/\lambda_I \longrightarrow 0$, as can be seen from Eqs. (5) and (15):

Here it shows clearly that if $p_I << (1/30) \sqrt{\frac{m}{2\pi k T_I}} (R_1/\lambda_I) \ln (R_2/R_1)$,

Knudsen's formula is quite applicable.

On the other hand, Eqs. (5) and (15) readily yield the Fourier result

$$q_R = \frac{k_c (T_I - T_{II})}{R \ln (R_2/R_1)} ,$$

as soon as we set $(R_1/\lambda_I) \longrightarrow \infty$ and utilize the relation $k_c = (15/4)\mu_c$ (R/m) for a monatomic gas.

Calculation of heat loss over the whole range of densities has also been done by Ai using Grad's thirteen moment equations[23]. Ai obtains the Fourier heat conduction relation over the whole range of densities and gives the result

$$\frac{Q}{Q_\infty} = \frac{1}{1 + \frac{1}{(8/15) (R_1/\lambda_I) \ln (R_2/R_1)}} , \qquad (19)$$

which yields a value twice as large as the actual heat loss at low pressures. As one can learn from Knudsen's formula [Eq. (18)] the heat loss at low pressures is proportional to the difference between the gas temperature and the temperature of the solid wall. Grad's formulation lacks the "two-sided" angular effect, and always overestimates the temperature difference at the wall by a factor of two. (See next section.) The same factor is found when the Fourier relation is used in conjunction with the Maxwell-Smoluchowski temperature "jump" condition (Section I).

Numerous experiments have been performed using the heated fine wire to determine gaseous conductivity, or mostly thermal accommodation coefficients. Conductivity measurements are often made at normal density with different temperatures, while the determinations of accommodation coefficients are usually done at low pressures. Unfortunately, data obtained in the past years are utterly inconsistent. Values of accommodation coefficient for a given pressure differing from each other by one or two orders of magnitude are not surprising at all. Hartnett[16] in his survey report on accommodation coefficients attributes this discrepancy to three factors: (1) the properties of the solid surface which are usually unspecified greatly affect the result; (2) evaluation of the accommodation coefficient by Knudsen's formula [Eq. (18)] for free molecular flow is often unjustified, because the pressures are usually not low enough to insure the free molecular limit; (3) use of an excessive radiation correction. Besides the

inconsistency of these experiments, most publications give only the final accommodation coefficients; a backward deduction to the heat loss is not only dangerous but also impossible owing to lack of knowledge of some physical constants employed in their computations.

The most recent measurement designed solely to study conductive heat transfer is done by Bomelburg[17]. He uses Wollaston wire of diameters 1.25μ, 5μ, 10μ, and bell jars of diameters 4 inches and 10 inches*. His results in the transition regime are reproduced in Figure 3, in which the three curves represent calculations according to Eq. (16). It is understandable that at low pressures when $\lambda_I > 200\ R_1$, radiation and end losses become dominant; thus conduction measurements at this range would be more difficult. But it has been clearly shown in Figure 3 that Bomelburg's experiment agrees with Eq. (16) fairly well.

Tracing back chronologically, we find the measurements by Schäfer, Rating, and Eucken in 1942[2]. They use a platinum wire of $R_1 = 0.00208$ cm in a tube of inner radius $R_2 = 0.294$ cm. Tests are run at 3.5°C with pressures ranging from 1. to 1/3000 atmosphere, so the ratio $2R_1/\lambda_I$ covers a wide region from 0.1 to 1000. Heat transfer results for Argon and CO_2 are plotted in Figure 4, in which the solid curve again represents Eq. (16). Points for Argon all fall along the predicted curve with a maximum deviation of 10 per cent at the lowest pressure point. The fact that the heat loss for CO_2 also obeys Eq. (16) is rather amazing, as the experiments of CO_2 went down to as low as 1/200 atm. One would expect that the factor 4/15 in Eq. (16) supposingly valid for monatomic gas only should be modified for a polyatomic gas. However, the general agreement is certainly not accidental.

Other experiments giving heat loss data have all been performed with diatomic gases. Gregory and Archer's measurement[3] (1926) using air and hydrogen gave Q/Q_∞ values 20 - 30 per cent lower than that predicted by Eq. (16). Knudsen's classical experiment[18] (1911) also using hydrogen at various pressure levels showed higher heat loss than expected. Fredlund[12] later correlated Knudsen's data with a formula exactly like Eq. (16). His correlation required a numerical factor approximately three times larger than 4/15, which is qualitatively in the right direction for a polyatomic gas correction.

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* Private communication through D. K. Ai.

IV. 2. Temperature Distribution

The expression for the mean gas temperature [Eq. (17)] is rewritten here

$$\frac{1 - \overline{T}}{1 - \overline{T}_{II}} = \delta \left[\tfrac{1}{2} + (1/\pi) \cos^{-1} (R_1/R) \right] + (1-\delta) \frac{\ln (R/R_1)}{\ln (R_2/R_1)} \qquad (20)$$

where

$$\delta = \left[1 + (4/15) (R_1/\lambda_I) \ln (R_2/R_1) \right]^{-1} ,$$

the significance of this parameter will be clear shortly. This expression shows that the temperature field is composed of two parts (See Figure 5.): the first (part 1) is an angular part weighted by δ, the second(part 2) is logarithmic, and it is in turn weighted by the quantity $(1-\delta)$. Part 1 has the character of a free molecular temperature field, while the other part has the same character as the Fourier solution. One should note that $\cos^{-1} (R_1/R)$ becomes practically equal to $\pi/2$ at about ten diameters from the center of the inner cylinder. Thus, at fairly low pressures the physical presence of the wire has no influence on the temperature at a point several diameters away. As δ becomes smaller, the logarithmic part would penetrate deeper from R_2 towards R_1 , and it finally dominates the whole temperature field when $\delta \longrightarrow 0$. A sketch of the temperature distribution for various values of $2\lambda_I/R_1$ is given in Figure 6.

From another point of view, the temperature distribution can be interpreted as the composition of an "outer" solution and an "inner" solution. The "outer" solution describes the temperature field corresponding to the solution of the Fourier heat conduction problem with a temperature "jump" at the inner wire when $R_2/R_1 >> 1$, which is the only case in which this "splitting" makes sense. This field is given by the expression

$$\left(\frac{1 - \overline{T}}{1 - \overline{T}_{II}} \right)_{outer} = \delta \left[1 + (4/15) (R_1/\lambda_I) \ln (R_2/R_1) \right] .$$

The inner solution is of the form

$$C \left[\tfrac{1}{2} + (1/\pi) \cos^{-1} (R_1/R) \right] ,$$

where the quantity in brackets is exactly the free-molecule solution, and C is an undetermined constant. Now, if one matches the inner and outer solutions by requiring that

$$\lim_{\overline{R}\to\infty}\left(\frac{1-\overline{T}}{1-\overline{T}_{II}}\right)_{inner} = \lim_{\overline{R}\to 1}\left(\frac{1-\overline{T}}{1-\overline{T}_{II}}\right)_{outer} ,$$

then $C = \delta$. Evidently Eq. (20) represents the full solution which is valid over the whole region.

Temperature jump phenomenon is solely accounted for by the angular part (part 1), which contributes to $(1 - \overline{T})/(1 - \overline{T}_{II})$ a difference of $\delta/2$ at the surface of the wire [Eq. (20)] . So the gas temperature at the free molecular limit equals the algebraic mean of T_I and T_{II} as one would expect in this linearized case. Grad's scheme employing only the logarithmic term gives a temperature profile

$$\frac{1-\overline{T}}{1-\overline{T}_{II}} = \delta + (1-\delta)\frac{\ln(R/R_1)}{\ln(R_2/R_1)} \quad * \quad , \tag{21}$$

from which one finds the "temperature jump" at the wire surface to be δ, which differs from the present result by a factor of two, thus over-estimating the heat loss by the same factor. Moreover, Eq. (21) yields a minimum (or maximum) somewhere in the annulus and is clearly physically unrealistic. The importance of the two-sided angular effect is fully shown here. On the other hand, at the wall of the outer cylinder the gas temperature there is practically the same as the wall temperature T_{II} for any value of R_1/λ_I if $R_2 > 20R_1$. In that case, as far as the outer cylinder is concerned, there is virtually no free molecular limit regardless how large the mean free path becomes.

At the surface of the hot wire there exists a thin region usually known as a Knudsen layer at small degrees of rarefaction where the angular part (part 1; Figure 5) effectively influences the temperature profile. This layer can be brought out by considering that

$$\text{at } \overline{R}_\ell = 1 + \Delta \quad \text{with} \quad \Delta << 1 ,$$

$$\cos\alpha = 1 - (\alpha^2/2) + \ldots = 1/\overline{R}_\ell = 1 - \Delta + \ldots$$

$$\alpha = (2\Delta)^{\frac{1}{2}} = \left[2(\overline{R}_\ell - 1)\right]^{\frac{1}{2}}$$

and

$$\ln \overline{R}_\ell = \Delta = \overline{R}_\ell - 1 ,$$

so

---

* This expression though not given explicitly in Reference 23 can be deduced from it easily.

$$1/\delta \left(\frac{1 - \overline{T}}{1 - \overline{T}_{II}} \right) \simeq \tfrac{1}{2} + (1/\pi)\sqrt{2(\overline{R}_\ell - 1)} + (4/15)(R_1/\lambda_I)(\overline{R}_\ell - 1) \quad (22)$$

Figure 7 sketches the separate terms of Eq. (22). The layer is then defined by the region where the angular and logarithmic parts are of the same order of magnitude, namely,

$$(1/\pi)\sqrt{2(\overline{R}_\ell - 1)} \simeq (4/15)(R_1/\lambda_I)(\overline{R}_\ell - 1)$$

or

$$\overline{R}_\ell - 1 \simeq 2.86\,(\lambda_I/R_1)^2 \quad ,$$

when $(\lambda_I/R_1) << 1$.

Thus the thickness of this Knudsen layer is proportional to the square of the Knudsen number.

A comparison between predicted and experimental temperature distributions has not been possible, because up to the present time, the only temperature distribution measurements in rarefied gases were done by Lazareff[19], Mandell, and West[20] between two parallel plates. Experiments designed to chart temperatures between two concentric cylinders have not yet been initiated.

IV. 3. "Free Molecular" Criterion

The minimum size of the mean free path required to insure that free molecular conditions prevail in an experiment has always been a puzzling question. The choice between conditions like $\lambda_I >> R_2 > R_1$ or $R_2 > \lambda_I >> R_1$ is quite uncertain. The confusion can be totally avoided by considering the quantity δ. One realizes that δ is in fact the true criterion for free molecular flow; neither R_1/λ_I nor R_2/R_1 alone governs the situation; i. e., $\delta \longrightarrow 1$ signifies the free molecular limit, while $\delta \longrightarrow 0$ represents the continuum regime. For instance, with an apparatus of given R_2/R_1 , $\delta \longrightarrow 1$ can be reached by reducing the gas density; or at a given gas condition, one achieves free molecular flow by increasing the R_2/R_1 ratio.

Referring to the definition of δ , one can now safely impose numerically that

$$(R_1/\lambda_I)\ \ln(R_2/R_1) \leqslant (1/10)$$

or equivalently

$$\frac{\lambda_I}{R_2 - R_1} \geqslant \frac{10 \ \ln (R_2/R_1)}{(R_2/R_1) - 1}$$

as the domain of free molecular flow, where Knudsen's formula [Eq. (18)] is valid within about 3 per cent. On the other hand, the condition

$$(4/15)(R_1/\lambda_I) \ \ln (R_2/R_1) \leqslant 20. \text{ (say)}$$

or

$$\frac{\lambda_I}{R_2 - R_1} \leqslant (1/75) \cdot \frac{\ln (R_2/R_1)}{(R_2/R_1) - 1}$$

represents the continuum limit where the Fourier result will be correct within 5 per cent. Figure 8 shows these two domains as well as the transition region for different values of R_2/R_1 .

IV. 4. Fourier-Maxwell-Smoluchowski Formulation

It has been generally accepted that the Fourier relation with the Maxwell-Smoluchowski temperature jump boundary condition would be fairly correct for gases of small degree of rarefaction. Its limit of validity for a problem involving curvatures has never been investigated. According to the relation

$$q_R = - k_c \, (dT/dR)$$

and the boundary conditions that

$$T_I - T(R_1) = - (15/8) \ \lambda_I \ (dT/dR)_{R=R_1}$$

and

$$T(R_2) - T_{II} = - (15/8) \ \lambda_I \, (dT/dR)_{R=R_2}$$

one would obtain an expression equivalent to Eq. (20) for the temperature distribution, namely,

$$\frac{1 - \overline{T}}{1 - \overline{T}_{II}} = \delta' + \left[1 - \left(1 + \frac{R_1}{R_2} \right) \delta' \right] \frac{\ln (R/R_1)}{\ln (R_2/R_1)} \qquad (23)$$

with

$$\delta' = \left[\left(1 + \frac{R_1}{R_2} \right) + (8/15) \, (R_1/\lambda_I) \ \ln (R_2/R_1) \right]^{-1} .$$

The ratio of heat transfer to the heat transfer in the limit $\lambda_I/R_1 \longrightarrow 0$ is

$$\frac{Q}{Q_\infty} = \frac{q_R}{q_{R_\infty}} = \frac{1}{1 + \dfrac{1 + (R_1/R_2)}{(8/15)\cdot(R_1/\lambda_I)\ \ell n\,(R_2/R_1)}} \quad . \tag{24}$$

In the case when $R_2 \simeq R_1$, namely, the gap between two cylinders is small in comparison with R_1, the curvature effect then is not important and Eqs. (23) and (24) would be quite correct even at low pressures. This situation is not surprising, as we have learned from the linearized case

$$\left[\frac{T_I - T_{II}}{T_I} << 1 \right]$$

of the plane Couette flow problem[15, 24]. There the Fourier-Maxwell-Smoluchowski result can be valid even when λ_I is large, at least with the minimum number of moments. However, the present type of apparatus normally has a large R_2/R_1 ratio; therefore, the heat transfer predicted by the Smoluchowski method [Eq. (19)] overshoots by a factor of two when $\lambda_I \geq R_1$, exactly as in Ai's result*. The present temperature profile [Eq. (23)] differs from Ai's [Eq. (21)] appreciably, because $\delta \neq \delta'$ in general. Now, one may confidently confirm a long time belief that, as far as gross quantities (like heat flux, total drag) are concerned, the Navier-Stokes-Fourier relations along with velocity-slip or temperature-jump boundary conditions would be fairly good for a linearized problem in which all curvature effects can be considered negligible, but details (like velocity or temperature profile) so obtained would be open to doubt.

The domain of validity of the Fourier-Maxwell-Smoluchowski formulation for this "hot-wire" instrument now can be estimated from Eqs. (23) and (24) as

$$\lambda_I/R_1 << (8/15)\ \ell n\,(R_2/R_1) \qquad \text{if } (R_2/R_1) >> 1$$

or numerically (say)

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* Ai in his study has imposed a boundary condition $T(R_2) = T_{II}$ at all density levels, which would be true only if $R_2 > 20\ R_1$ [See Section IV. 2.], so his solution contains an implicit assumption of $R_2/R_1 >> 1$.

$$\lambda_I/R_1 < (8/150) \ln (R_2/R_1) \quad .$$

A sketch of the $q_R / - k_c (dT/dR)$ ratio [Figure 9] based on Eqs. (5) and (15) shows that Fourier's "law" is valid either far from the center wire at any density, or everywhere at normal density. One interesting note is that the ratio falls to zero at the wire surface for finite λ_I; this behavior arises owing to the infinite temperature gradient resulting from differentiation of $\cos^{-1} (R_1/R)$. It is quite similar to the situation of having an infinite velocity gradient at the forward stagnation point of a cylinder in rarefied gas flow. The gradient becomes finite if the curvature becomes small and the cylinder is transformed to a flat disk[25].

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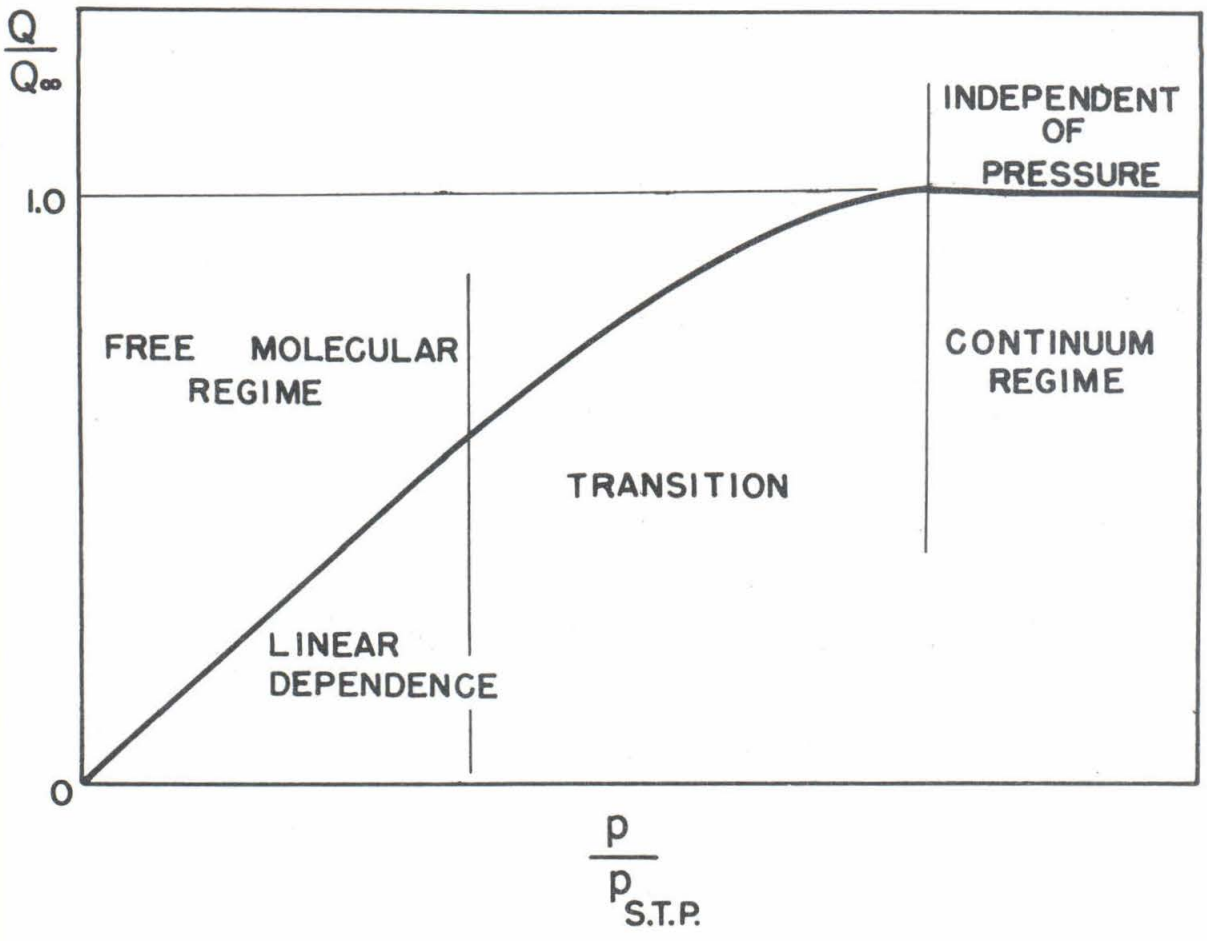


FIG. 1

VARIATION OF HEAT TRANSFER WITH GAS PRESSURE

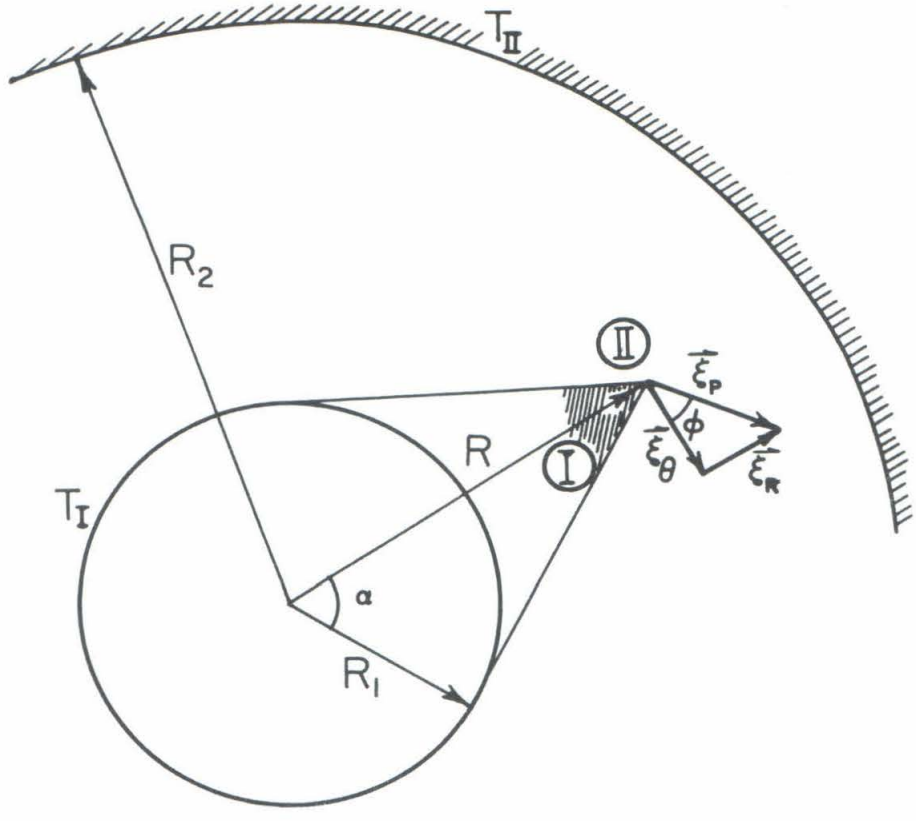


FIG. 2 CONFIGURATION IN CYLINDRICAL COORDINATES

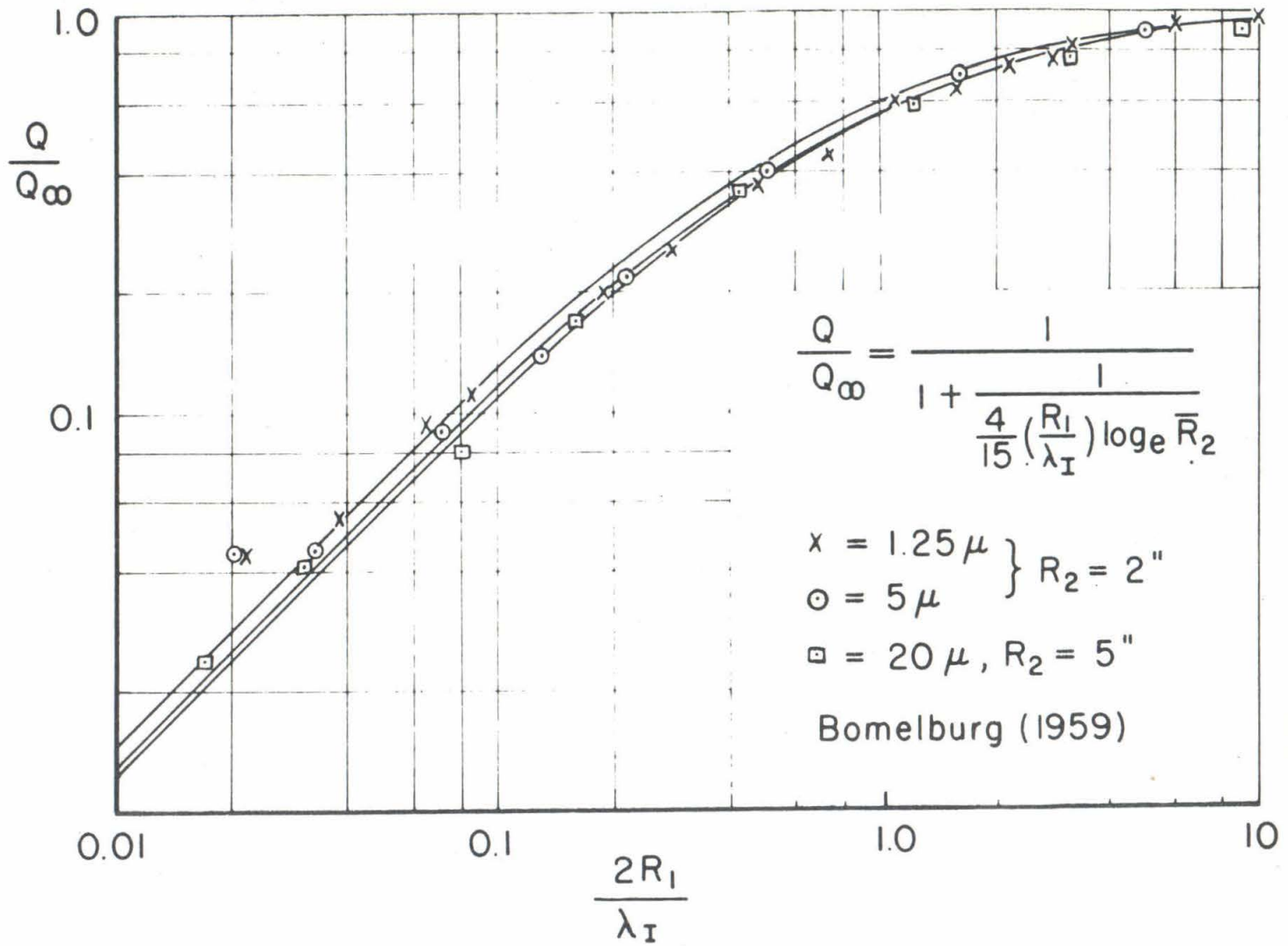


FIG. 3 COMPARISON WITH EXPERIMENT (REF. 17)

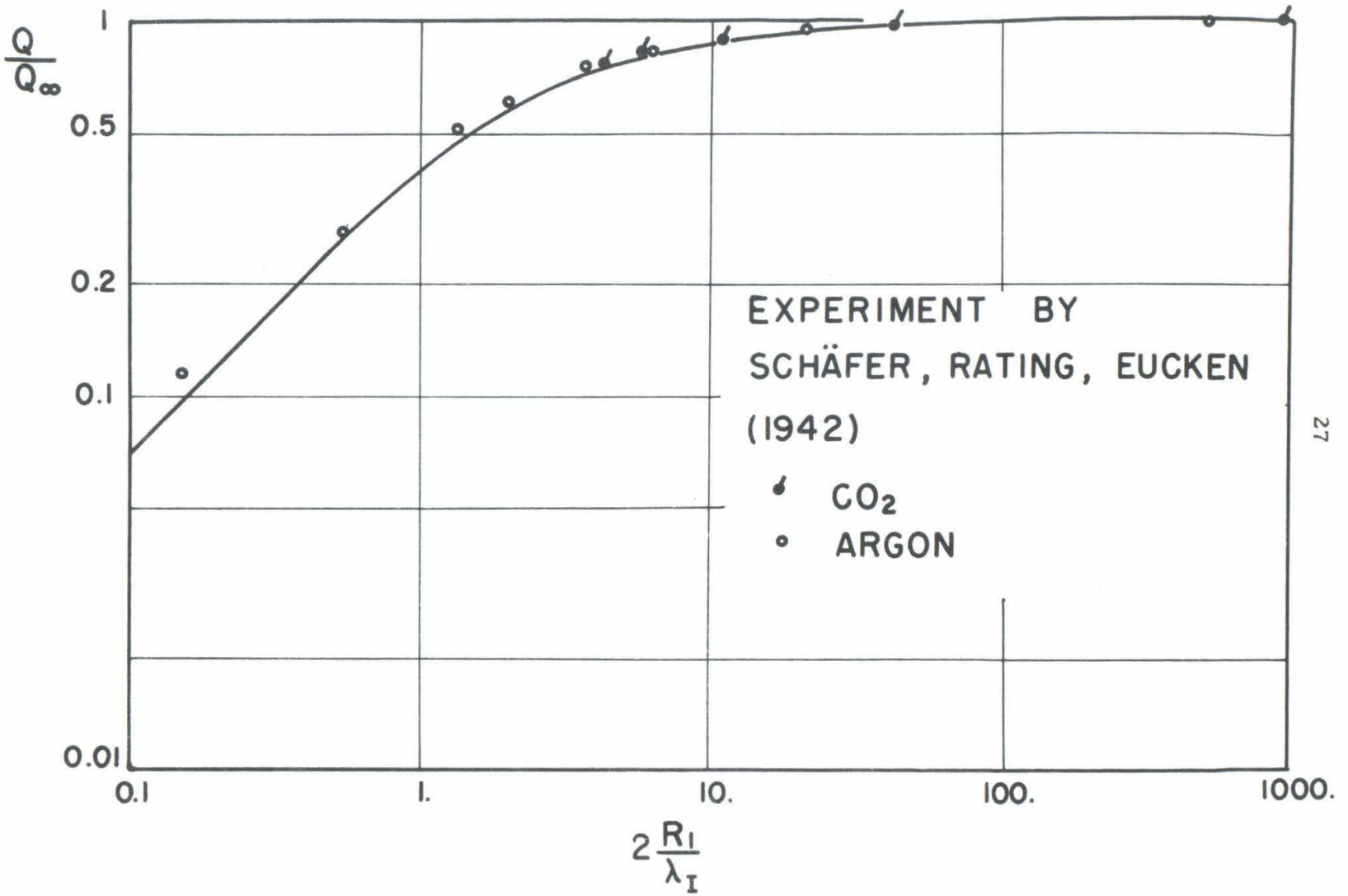


FIG. 4. COMPARISON WITH EXPERIMENT (REF. 2)

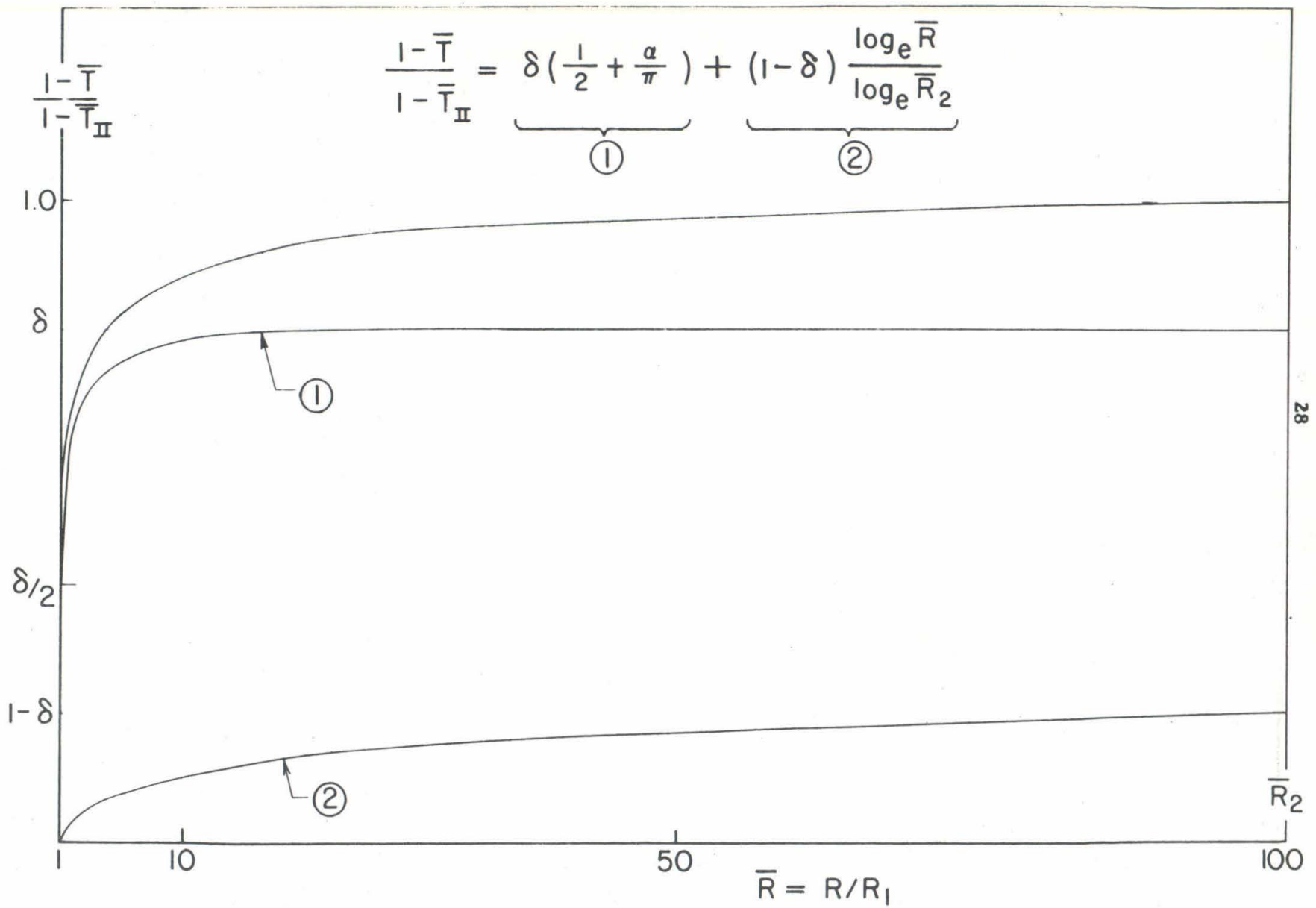


FIG. 5 TEMPERATURE DISTRIBUTION BETWEEN TWO CYLINDERS

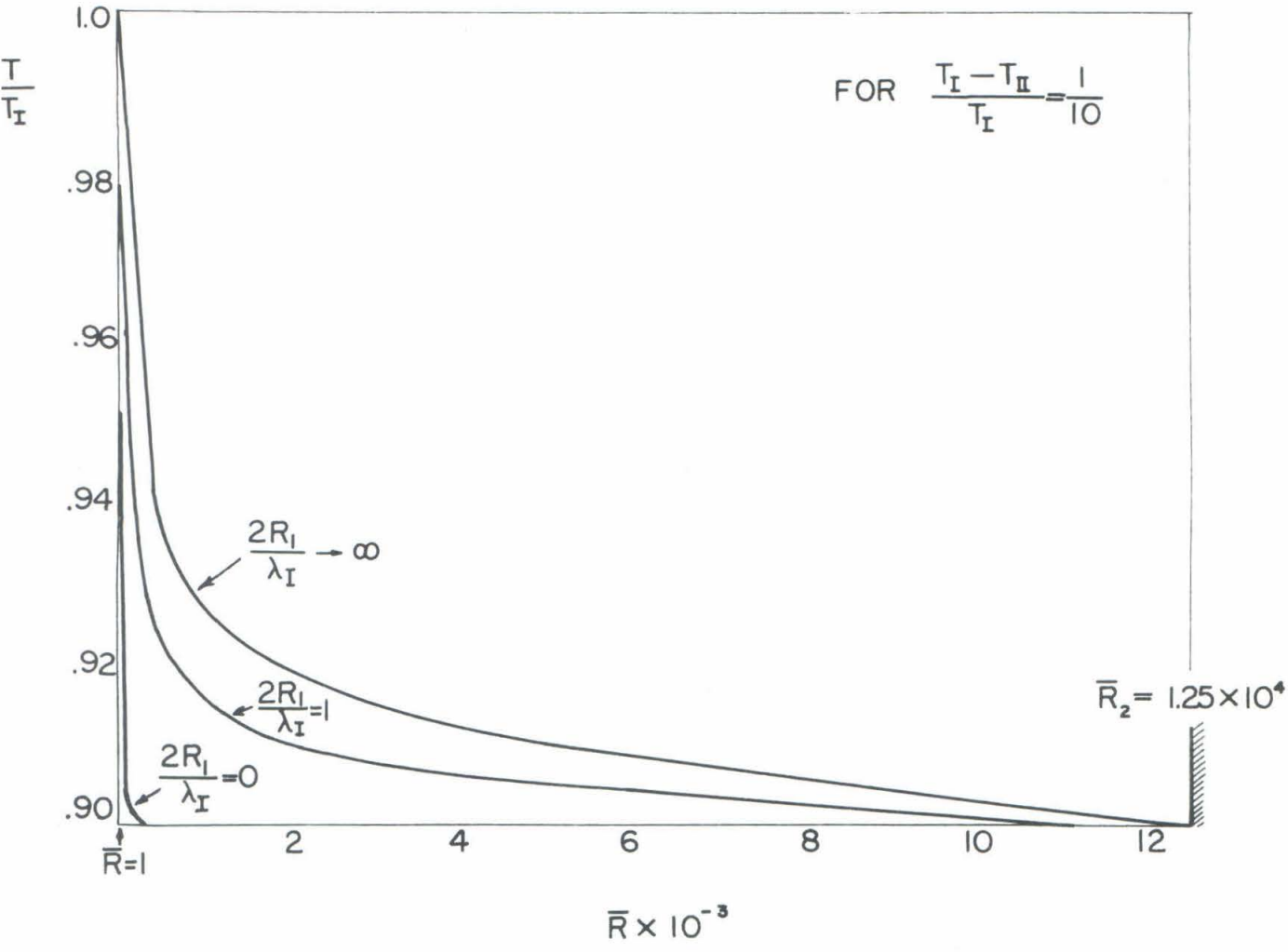


FIG. 6

TEMPERATURE DISTRIBUTION FOR VARIOUS VALUES OF $2R_1/\lambda_I$

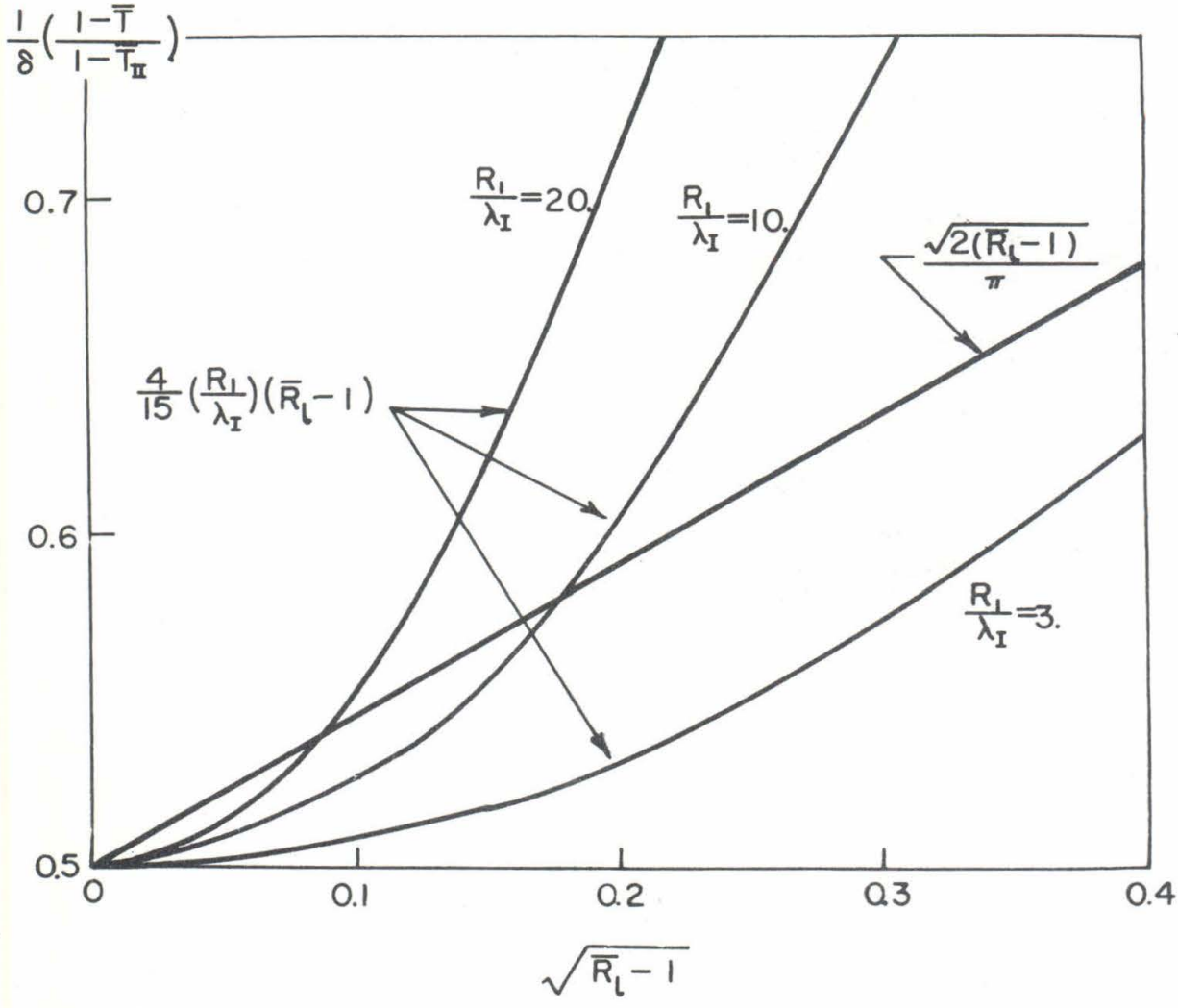


FIG. 7

THICKNESS OF KNUDSEN LAYER

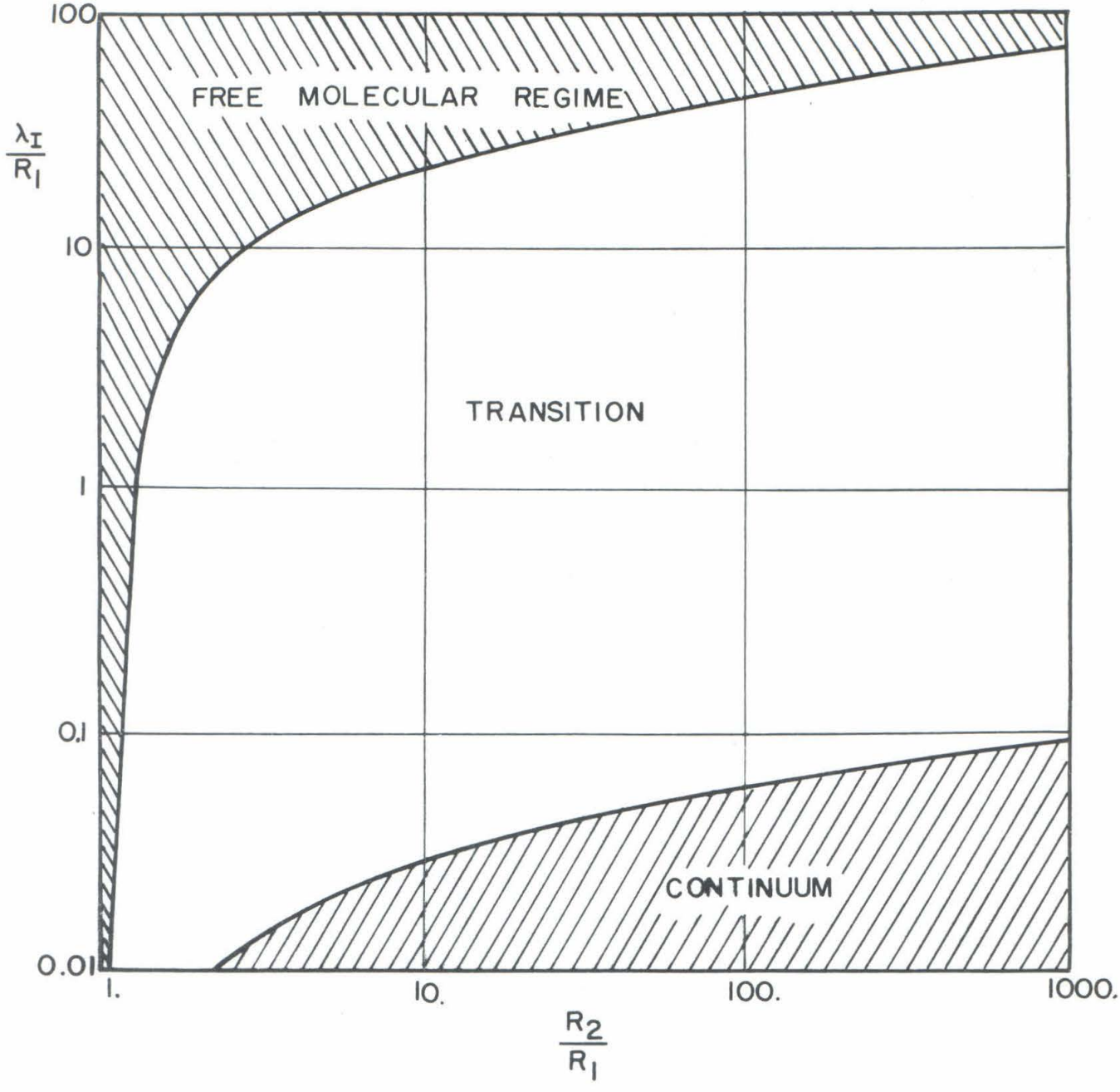
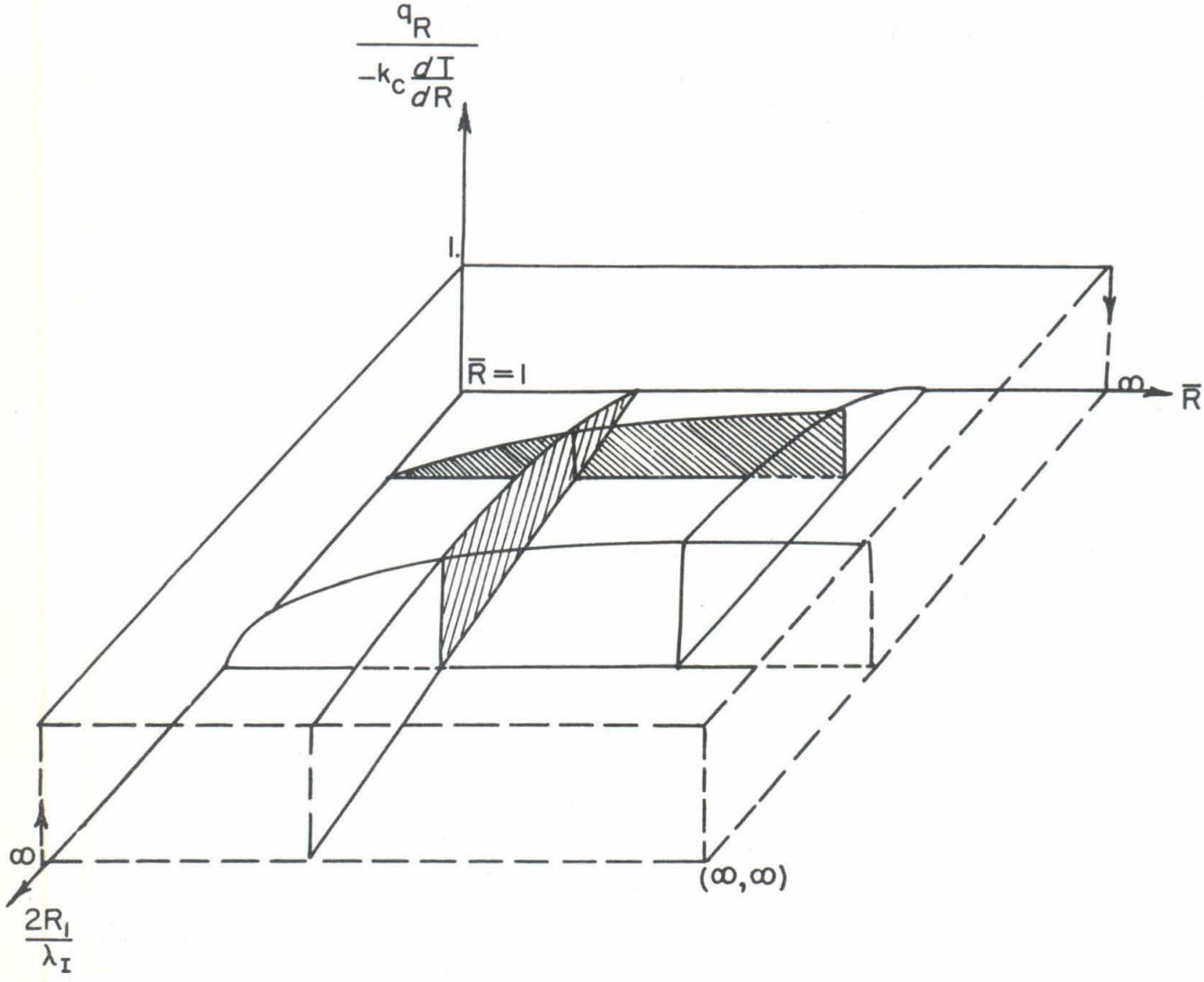


FIG. 8
CLASSIFICATION OF DENSITY REGIMES



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